MCC-003-1162004 Seat No. ______

M. Sc. (Sem. II) (CBCS) Examination

April / May - 2018

Mathematics - 2004

(Methods in Partial Differential Equation)

Faculty Code : 003
Subject Code : 1162004

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) There are 5 questions.
(2) All questions are compulsory.
(3) Each questions carries 14 marks.

1 Do as directed : (Each question carries two marks) **14**

(a) Find the complete integral of $zpq = p + q$.

(b) Find the integral curves of the equation
$$\frac{dx}{y^2} = \frac{dy}{-xy} = \frac{dz}{x(z-2y)}$$

(c) Solve the equation $r + s - 2t - p - 2q = 0$.

(d) Check whether the p.d.e. $3y(a-z)\,dy = \left(y - z^2 + \left(a^2 + y\right)\right) dz$ is integrable or not ?

(e) If $z = f(x+ky) - g(x-ky)$ where f and g are arbitrary functions and k is a constant then show that $z_{yyy} = k^2 z_{xx}$.

(f) Find the equation of a tangent plane to the surface $y^3 + x^2 + 3yz - z^3 = 6$ at point $(-2, -1, -3)$.

(g) Verify the equation is exact or not
$$(yz)\,dx + \left(x^2 z - xy\right) dz + \left(x^2 y - xz\right) dy = 0.$$

2 Answer any **two** of the following : **2×7=14**

(a) Solve using Nattani's method $\left(y^2 + z^2\right) dx + xydy + xzdz = 0$.

(b) Classify the equation and convert it into canonical form $y^2 r - p = x^2 t - q$.

(c) If $(\beta D'+\gamma)^2$ with $\alpha \neq 0$ is a factor of $F(D, D')$, then a solution of the equation $F(D, D')$ is,

$$z = e^{\frac{-\gamma}{\beta}y}\left(\phi_1(\beta x) + y\,\phi_2(\beta x)\right)$$

Where $\phi_i = \phi_i(\varepsilon)$ is an arbitrary function of a single variable $(i = 1, 2)$.

3 All are compulsory : **14**

(a) Find the integral curves of the equation **4**

$$\frac{dx}{\left(x^3+3xy^2\right)} = \frac{dy}{\left(y^3+3yx^2\right)} = \frac{dz}{(2z)\left(x^2+y^2\right)}.$$

(b) Solve using homogenous method **5**

$$yz(y+z)\,dx + xy(x+y)\,dz + xz(x+z)\,dy = 0$$

(c) Find the orthogonal trajectory on the cone $x^2 + y^2 = z^2 \tan^2 \alpha$ of its intersection with family of plane parallel to $z = c$. **5**

OR

3 All are compulsory : **14**

(a) Using Charpit's method solve $(z+qy)^2 = p$. **5**

(b) Solve $\left(3D^2 + 8DD' + 4D'^2\right)z = e^{y-2x} + e^{x-y}$. **4**

(c) Find the surface which intersects the surface of the system $z(x+y) = c(3z+1)$ orthogonally and which passes through the circle $x^2 + y^2 = 1, z = 1$. **5**

4 Answer any two of the following : **2×7=14**

(a) Prove that for any non-zero functions $\mu = \mu(x, y, z)$ and $X = (P, Q, R)$ where P, Q, R are the functions of x, y, z then

$$X \cdot Curl\, X = 0 \text{ iff } (\mu X) \cdot Curl\,(\mu X) = 0.$$

(b) Prove that a pfaffian differential equation

$\left(y^2 + yz\right)dx + \left(xz + z^2\right)dy = -\left(-yx + y^2\right)dz$ is integrable.

Also find the complete primitive.

(c) Show that the complete integral of the equation $f\left(u_x, u_y, u_z\right) = 0$ is $u = ax + by + g(a, b)\,z + c$ where a, b, c are constants. Also find the complete integral of the equation $u_x \cdot u_x \cdot u_z = u_x + u_y + u_z$.

5 Answer any **two** of the following : **2×7=14**

(a) (i) Solve $f\left(x + y, x - \sqrt{z}\right)$.

(ii) Using Jacobi's Method Solve $xyp = q$.

(b) Find the System of orthogonal trajectories on the sphere $x^2 + y^2 + z^2 = a^2$ of its intersection with the paraboloids $xy = cz$.

(c) Find the general solution of

$\left(-8D'^2 + 2DD' + D^2\right) z = (2x + 3y)^{\frac{1}{2}}$ using general method.
